CH6 – LINEAR INEQUALITIES

Exercise 6.1 Page No: 122

1. Solve 24x < 100, when
(i) x is a natural number.
(ii) x is an integer.
Solution:
(i) Given that 24x < 100
Now we have to divide the inequality by 24 then we get x < 25/6
Now when x is a natural integer then
It is clear that the only natural number less than 25/6 are 1, 2, 3, 4.
Thus, 1, 2, 3, 4 will be the solution of the given inequality when x is a natural number.
Hence {1, 2, 3, 4} is the solution set.
(ii) Given that 24x < 100
Now we have to divide the inequality by 24 then we get x < 25/6
now when x is an integer then
It is clear that the integer number less than 25/6 are…-1, 0, 1, 2, 3, 4.
Thus, solution of 24 x < 100 are…,-1, 0, 1, 2, 3, 4, when x is an integer.
Hence {…, -1, 0, 1, 2, 3, 4} is the solution set.
2. Solve – 12x > 30, when
(i) x is a natural number.
(ii) x is an integer.
Solution:
(i) Given that, – 12x > 30
Now, by dividing the inequality by -12 on both sides we get, x < -5/2
When x is a natural integer then
It is clear that there is no natural number less than -2/5 because -5/2 is a negative number and natural numbers are positive numbers.
Therefore there would be no solution of the given inequality when x is a natural number.
(ii) Given that, – 12x > 30
Now by dividing the inequality by -12 on both sides we get, x < -5/2
When x is an integer then
It is clear that the integer number less than -5/2 are…, -5, -4, – 3
Thus, solution of – 12x > 30 is …,-5, -4, -3, when x is an integer.
Therefore the solution set is {…, -5, -4, -3}

3. Solve $5x - 3 < 7$, when
(i) x is an integer
(ii) x is a real number
Solution:
(i) Given that, $5x - 3 < 7$
Now by adding 3 on both sides, we get,
$5x - 3 + 3 < 7 + 3$
The above inequality becomes
$5x < 10$
Again, by dividing both sides by 5 we get,
$5x/5 < 10/5$
$x < 2$
When x is an integer, then
It is clear that that the integer number less than 2 are…, -2, -1, 0, 1.
Thus, solution of $5x - 3 < 7$ is …,-2, -1, 0, 1, when x is an integer.
Therefore the solution set is {…, -2, -1, 0, 1}
(ii) Given that, $5x - 3 < 7$
Now by adding 3 on both sides, we get,
$5x - 3 + 3 < 7 + 3$
Above inequality becomes
$5x < 10$
Again, by dividing both sides by 5, we get,
$5x/5 < 10/5$
$x < 2$
When x is a real number, then
It is clear that the solutions of $5x - 3 < 7$ will be given by $x < 2$ which states that all the real numbers that are less than 2.
Hence the solution set is $x \in (-\infty, 2)$
4. Solve $3x + 8 > 2$, when
(i) x is an integer.
(ii) x is a real number.
Solution:
(i) Given that, $3x + 8 > 2$
Now by subtracting 8 from both sides, we get,
$3x + 8 - 8 > 2 - 8$
The above inequality becomes,
$3x > -6$

Again by dividing both sides by 3, we get,
$3x/3 > -6/3$
Hence $x > -2$
When x is an integer, then
It is clear that the integer numbers greater than -2 are -1, 0, 1, 2,…
Thus, solution of $3x + 8 > 2$ is -1, 0, 1, 2,… when x is an integer.
Hence the solution set is {-1, 0, 1, 2,…}
(ii) Given that, $3x + 8 > 2$
Now by subtracting 8 from both sides we get,
$3x + 8 – 8 > 2 – 8$
The above inequality becomes,
$3x > – 6$
Again, by dividing both sides by 3, we get,
$3x/3 > -6/3$
Hence $x > -2$
When x is a real number.
It is clear that the solutions of $3x + 8 >2$ will be given by $x > -2$ which means all the real numbers that are greater than -2.
Therefore the solution set is $x \in (-2, \infty)$

Solve the inequalities in Exercises 5 to 16 for real x.

5. $4x + 3 < 5x + 7$

Solution:

Given that, $4x + 3 < 5x + 7$
Now by subtracting 7 from both the sides, we get
$4x + 3 – 7 < 5x + 7 – 7$
The above inequality becomes,
$4x – 4 < 5x$
Again, by subtracting 4x from both the sides,
$4x – 4 – 4x < 5x – 4x$
$x > – 4$
∴The solutions of the given inequality are defined by all the real numbers greater than -4.
The required solution set is $(-4, \infty)$

6. $3x – 7 > 5x – 1$

Solution:

Given that,
$3x – 7 > 5x – 1$

Now, by adding 7 to both the sides, we get

$3x - 7 + 7 > 5x - 1 + 7$

$3x > 5x + 6$

Again, by subtracting 5x from both the sides,

$3x - 5x > 5x + 6 - 5x$

$-2x > 6$

Dividing both sides by -2 to simplify, we get

$-2x/-2 < 6/-2$

$x < -3$

∴ The solutions of the given inequality are defined by all the real numbers less than -3.

Hence the required solution set is $(-\infty, -3)$

7. $3(x - 1) \leq 2(x - 3)$

Solution:

Given that, $3(x - 1) \leq 2(x - 3)$

By multiplying, the above inequality can be written as

$3x - 3 \leq 2x - 6$

Now, by adding 3 to both the sides, we get

$3x - 3 + 3 \leq 2x - 6 + 3$

$3x \leq 2x - 3$

Again, by subtracting 2x from both the sides,

$3x - 2x \leq 2x - 3 - 2x$

$x \leq -3$

Therefore, the solutions of the given inequality are defined by all the real numbers less than or equal to -3.

Hence, the required solution set is $(-\infty, -3]$

8. $3(2 - x) \geq 2(1 - x)$

Solution:

Given that, $3(2 - x) \geq 2(1 - x)$

By multiplying, we get

$6 - 3x \geq 2 - 2x$

Now, by adding 2x to both the sides,

$6 - 3x + 2x \geq 2 - 2x + 2x$

$6 - x \geq 2$

Again, by subtracting 6 from both the sides, we get

$6 - x - 6 \geq 2 - 6$

$-x \geq -4$

Multiplying throughout inequality by negative sign, we get
$x \leq 4$
∴ The solutions of the given inequality are defined by all the real numbers greater than or equal to 4.
Hence the required solution set is (- ∞, 4]

9. $x + x/2 + x/3 < 11$

Solution:

Given that,

$$x + \frac{x}{2} + \frac{x}{3} < 11$$

By taking x as common then we get

$$x\left(1 + \frac{1}{2} + \frac{1}{3}\right) < 11$$

By taking LCM

$$x\left(\frac{6+3+2}{2}\right) < 11$$

$$\frac{11x}{6} < 11$$

$$x\left(\frac{6+3+2}{2}\right) < 11$$

$$\frac{11x}{6} < 11$$

Dividing by 11 on both sides,

$$\frac{11x}{6 \times 11} < \frac{11}{11}$$

$$\frac{x}{6} < 1$$

$x < 6$

The solutions of the given inequality are defined by all the real numbers less than 6.
Hence the solution set is (-∞, 6)

10. $x/3 > x/2 + 1$

Solution:

Given that,

$$\frac{x}{3} > \frac{x}{2} + 1$$

On rearranging and by taking LCM we get

$$\left(\frac{2x - 3x}{6}\right) > 1$$

$-x/6 > 1$

$-x > 6$

$x < -6$

∴ The solutions of the given inequality are defined by all the real numbers less than – 6.

Hence, the required solution set is (-∞, -6)

11. $3(x - 2)/5 \leq 5(2 - x)/3$

Solution:

Given that,

$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$

Now by cross-multiplying the denominators, we get

$9(x- 2) \leq 25 (2 - x)$

$9x - 18 \leq 50 - 25x$

Now adding 25x both the sides,

$9x - 18 + 25x \leq 50 - 25x + 25x$

$34x - 18 \leq 50$

Adding 25x both the sides,

$34x - 18 + 18 \leq 50 + 18$

$34x \leq 68$

Dividing both sides by 34,

$34x/34 \leq 68/34$

$x \leq 2$

The solutions of the given inequality are defined by all the real numbers less than or equal to 2.

Required solution set is (-∞, 2]

12. $\frac{1}{2}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6)$

Solution:

Given that,

$$\frac{1}{2}\left(\frac{3x}{5}+4\right) \geq \frac{1}{3}(x-6)$$

Now by cross - multiplying the denominators, we get

$$3\left(\frac{3x}{5}+4\right) \geq 2(x-6)$$

Multiplying by 3 we get

$$\left(\frac{9x}{5}+12\right) \geq 2x-12$$

On rearranging, we get

$$12+12 \geq 2x-\frac{9x}{5}$$

$$24 \geq \frac{10x-9x}{5}$$

$$24 \geq \frac{10x-9x}{5}$$

$$24 \geq \frac{x}{5}$$

$120 \geq x$

∴ The solutions of the given inequality are defined by all the real numbers less than or equal to 120.

Thus, (-∞, 120] is the required solution set.

13. 2 (2x + 3) – 10 < 6 (x – 2)

Solution:

Given that,

$2(2x + 3) - 10 < 6(x - 2)$

By multiplying, we get

$4x + 6 - 10 < 6x - 12$

On simplifying, we get

$4x - 4 < 6x - 12$

$4x - 6x < -12 + 4$

$-2x < -8$

Dividing by 2, we get;

$-x < -4$

Multiply by "-1" and change the sign.
$x > 4$
∴ The solutions of the given inequality are defined by all the real numbers greater than 4.
Hence, the required solution set is $(4, \infty)$.
14. $37 - (3x + 5) \geq 9x - 8(x - 3)$
Solution:
Given that, $37 - (3x + 5) \geq 9x - 8(x - 3)$
On simplifying, we get
$= 37 - 3x - 5 \geq 9x - 8x + 24$
$= 32 - 3x \geq x + 24$
On rearranging,
$= 32 - 24 \geq x + 3x$
$= 8 \geq 4x$
$= 2 \geq x$
All the real numbers of x which are less than or equal to 2 are the solutions of the given inequality
Hence, $(-\infty, 2]$ will be the solution for the given inequality
15. $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$
Solution:
Given,
$$\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5} = \frac{x}{4} < \frac{5(5x-2) - 3(7x-3)}{15}$$
On simplifying we get
$$= \frac{x}{4} < \frac{25x - 10 - 21x + 9}{15}$$
$$= \frac{x}{4} < \frac{4x-1}{15}$$
$= 15x < 4(4x - 1)$
$= 15x < 16x - 4$
$= 4 < x$
All the real numbers of x which are greater than 4 are the solutions of the given inequality
Hence, $(4, \infty)$ will be the solution for the given inequality

16. $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

Solution:

Given,

$\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5} = \frac{(2x-1)}{3} \geq \frac{5(3x-2) - 4(2-x)}{20}$

On rearranging we get

$= \frac{(2x-1)}{3} \geq \frac{15x - 10 - 8 + 4x}{20}$

$= \frac{(2x-1)}{3} \geq \frac{19x - 18}{20}$

$= \frac{(2x-1)}{3} \geq \frac{19x - 18}{20}$

$= 20(2x - 1) \geq 3(19x - 18)$

$= 40x - 20 \geq 57x - 54$

$= -20 + 54 \geq 57x - 40x$

$= 34 \geq 17x$

$= 2 \geq x$

∴ All the real numbers of x which are less than or equal to 2 are the solutions of the given inequality

Hence, (-∞, 2] will be the solution for the given inequality

Solve the inequalities in Exercises 17 to 20 and show the graph of the solution in each case on number line.

17. $3x - 2 < 2x + 1$

Solution:

Given,

$3x - 2 < 2x + 1$

Solving the given inequality, we get

$3x - 2 < 2x + 1$

$= 3x - 2x < 1 + 2$

$= x < 3$

Now, the graphical representation of the solution is as follows:

18. $5x - 3 \geq 3x - 5$

Solution:

We have,

$5x - 3 \geq 3x - 5$

Solving the given inequality, we get

$5x - 3 \geq 3x - 5$

On rearranging, we get

$= 5x - 3x \geq -5 + 3$

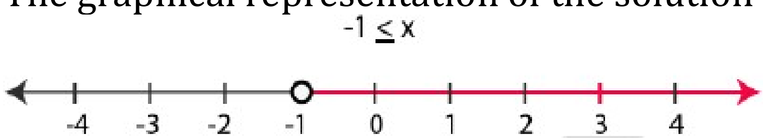
On simplifying,

$= 2x \geq -2$

Now, dividing by 2 on both sides, we get

$= x \geq -1$

The graphical representation of the solution is as follows:



19. $3(1 - x) < 2(x + 4)$

Solution:

Given,

$3(1 - x) < 2(x + 4)$

Solving the given inequality, we get

$3(1 - x) < 2(x + 4)$

Multiplying, we get

$= 3 - 3x < 2x + 8$

On rearranging, we get

$= 3 - 8 < 2x + 3x$

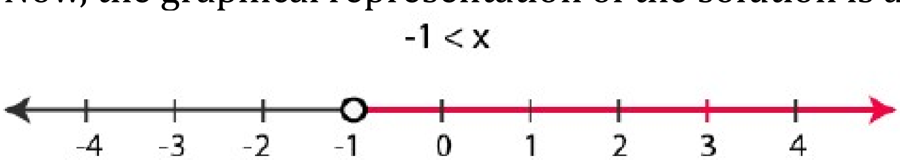
$= -5 < 5x$

Now by dividing 5 on both sides, we get

$-5/5 < 5x/5$

$= -1 < x$

Now, the graphical representation of the solution is as follows:



20. $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

Solution:

Given,

$$\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

Solving the given inequality, we get

$$\frac{x}{2} \geq \frac{5(5x-2)-3(7x-3)}{15}$$

On computing we get

$$= \frac{x}{2} \geq \frac{25x - 10 - 21x + 9}{15}$$

$$= \frac{x}{2} \geq \frac{4x-1}{15}$$

Solving the given inequality, we get

$$\frac{x}{2} \geq \frac{5(5x-2)-3(7x-3)}{15}$$

On computing we get

$$= \frac{x}{2} \geq \frac{25x - 10 - 21x + 9}{15}$$

$$= \frac{x}{2} \geq \frac{4x-1}{15}$$

On computing we get

$$= \frac{x}{2} \geq \frac{25x - 10 - 21x + 9}{15}$$

$$= \frac{x}{2} \geq \frac{4x-1}{15}$$

$= 15x \geq 2\,(4x - 1)$

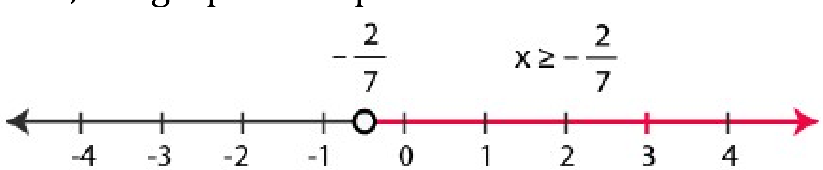
$= 15x \geq 8x - 2$

$= 15x - 8x \geq 8x - 2 - 8x$

$= 7x \geq -2$

$= x \geq -2/7$

Now, the graphical representation of the solution is as follows:



21. Ravi obtained 70 and 75 marks in the first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Solution:

Let us assume that x is the marks obtained by Ravi in his third unit test.

According to the question, all the students should have an average of at least 60 marks

$(70 + 75 + x)/3 \geq 60$

$= 145 + x \geq 180$

$= x \geq 180 - 145$

$= x \geq 35$

Hence, all the students must obtain 35 marks in order to have an average of at least 60 marks

22. To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in the first four examinations are 87, 92, 94 and 95, find the minimum marks that Sunita must obtain in the fifth examination to get Grade 'A' in the course.

Solution:

Let us assume Sunita scored x marks in her fifth examination

Now, according to the question, in order to receive A grade in the course, she must obtain an average of 90 marks or more in her five examinations

$(87 + 92 + 94 + 95 + x)/5 \geq 90$

$= (368 + x)/5 \geq 90$

$= 368 + x \geq 450$

$= x \geq 450 - 368$

$= x \geq 82$

Hence, she must obtain 82 or more marks in her fifth examination

23. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Solution:

Let us assume x to be the smaller of the two consecutive odd positive integers.

$\therefore$ The other integer is $= x + 2$

It is also given in the question that both the integers are smaller than 10.

$\therefore x + 2 < 10$

$x < 8$ … (i)

Also, it is given in the question that the sum of two integers is more than 11.

$\therefore x + (x + 2) > 11$

$2x + 2 > 11$

$x > 9/2$

$x > 4.5$ … (ii)

Thus, from (i) and (ii), we have,

x is an odd integer and it can take values 5 and 7.

Hence, possible pairs are (5, 7) and (7, 9)

24. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

Solution:

Let us assume x is the smaller of the two consecutive even positive integers.

$\therefore$ The other integer $= x + 2$

It is also given in the question that both the integers are larger than 5.

$\therefore x > 5$ ….(i)

Also, it is given in the question that the sum of two integers is less than 23.

$\therefore x + (x + 2) < 23$

$2x + 2 < 23$

$x < 21/2$

$x < 10.5$ … (ii)

Thus, from (i) and (ii) we have x is an even number and it can take values 6, 8 and 10.

Hence, possible pairs are (6, 8), (8, 10) and (10, 12).

25. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

Solution:

Let us assume the length of the shortest side of the triangle to be x cm.

$\therefore$ According to the question, the length of the longest side $= 3x$ cm

And, length of third side $= (3x - 2)$ cm

As, the least perimeter of the triangle $= 61$ cm

Thus, $x + 3x + (3x - 2)$ cm ≥ 61 cm

$= 7x - 2 \geq 61$

$= 7x \geq 63$

Now dividing by 7, we get

$= 7x/7 \geq 63/7$

$= x \geq 9$

Hence, the minimum length of the shortest side will be 9 cm.

26. A man wants to cut three lengths from a single piece of board of length 91cm. The second length is to be 3cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5cm longer than the second?

Solution:

Let us assume the length of the shortest piece to be x cm

∴ According to the question, length of the second piece = (x + 3) cm

And, length of third piece = 2x cm

As all the three lengths are to be cut from a single piece of board having a length of 91 cm

$\therefore x + (x + 3) + 2x \leq 91$ cm

$= 4x + 3 \leq 91$

$= 4x \leq 88$

$= 4x/4 \leq 88/4$

$= x \leq 22$ … (i)

Also, it is given in the question that, the third piece is at least 5 cm longer than the second piece.

$\therefore 2x \geq (x+3) + 5$

$2x \geq x + 8$

$x \geq 8$ … (ii)

Thus, from equation (i) and (ii), we have:

$8 \leq x \leq 22$

Hence, it is clear that the length of the shortest board is greater than or equal to 8 cm and less than or equal to 22 cm.

Exercise 6.2 Page No: 127

Solve the following inequalities graphically in two-dimensional plane:

1. $x + y < 5$

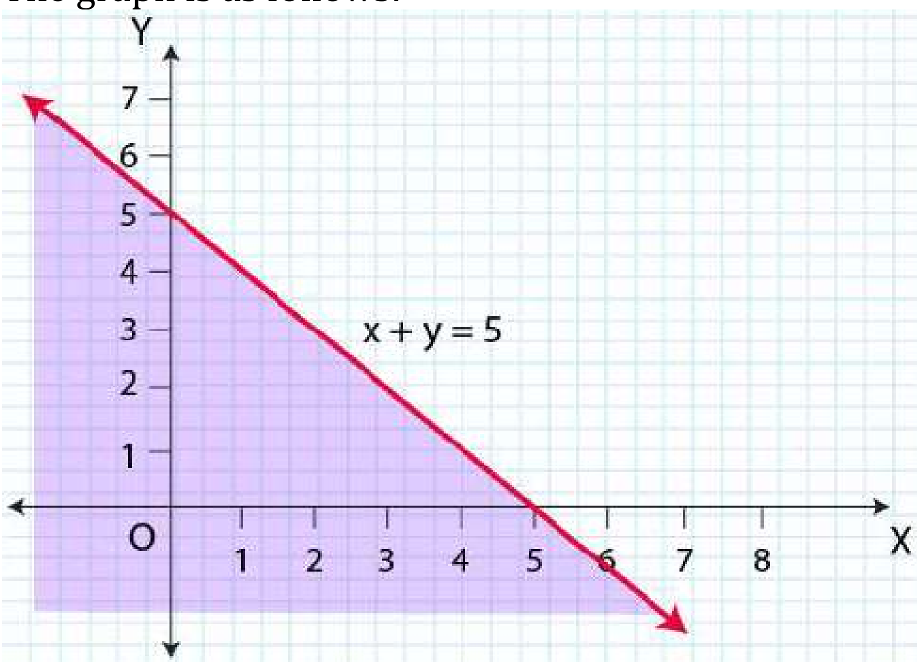
Solution:

Given $x + y < 5$

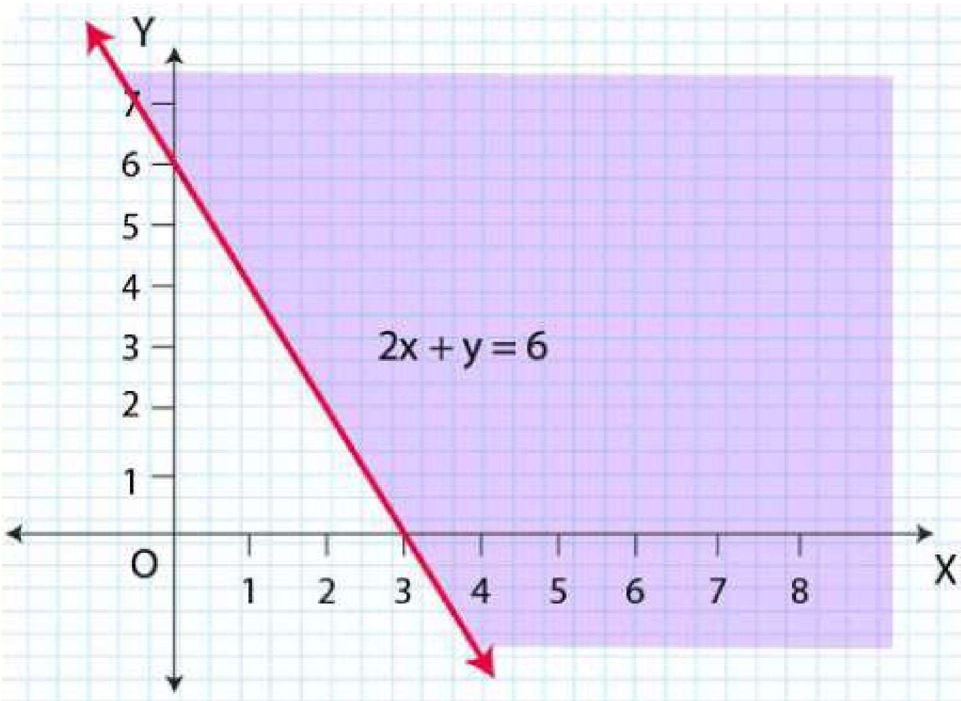
Consider

X	0	5
y	5	0

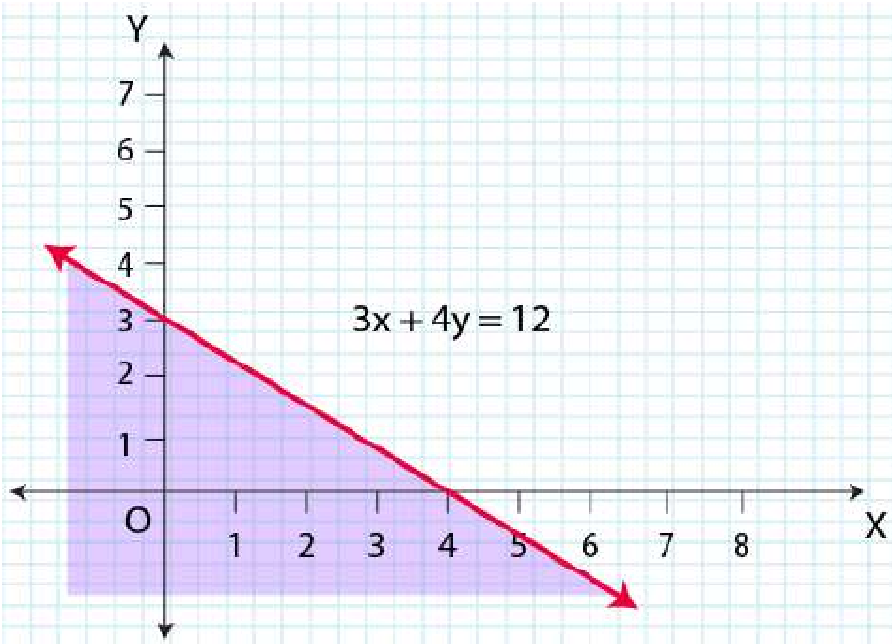
Now, draw a dotted line x + y = 5 in the graph (∵ x + y = 5 is excluded in the given question)
Now, consider x + y < 5
Select a point (0, 0)
⇒ 0 + 0 < 5
⇒ 0 < 5 (this is true)
∴ Solution region of the given inequality is below the line x + y = 5. (i.e., origin is included in the region)
The graph is as follows:



2. 2x + y ≥ 6
Solution:
Given 2x + y ≥ 6
Now, draw a solid line 2x + y = 6 in the graph (∵2x + y = 6 is included in the given question)
Now, consider 2x + y ≥6
Select a point (0, 0)
⇒ 2 × (0) + 0 ≥ 6
⇒ 0 ≥ 6 (this is false)
∴ Solution region of the given inequality is above the line 2x + y = 6. (away from the origin)
The graph is as follows:



3. $3x + 4y \leq 12$
Solution:
Given $3x + 4y \leq 12$
Now, draw a solid line $3x + 4y = 12$ in the graph ($\because 3x + 4y = 12$ is included in the given question)
Now, consider $3x + 4y \leq 12$
Select a point (0, 0)
$\Rightarrow 3 \times (0) + 4 \times (0) \leq 12$
$\Rightarrow 0 \leq 12$ (this is true)
$\therefore$ Solution region of the given inequality is below the line $3x + 4y = 12$. (i.e., origin is included in the region)
The graph is as follows:



4. $y + 8 \geq 2x$

Solution:

Given $y + 8 \geq 2x$

Now, draw a solid line $y + 8 = 2x$ in the graph ($\because y + 8 = 2x$ is included in the given question)

Now, consider $y + 8 \geq 2x$

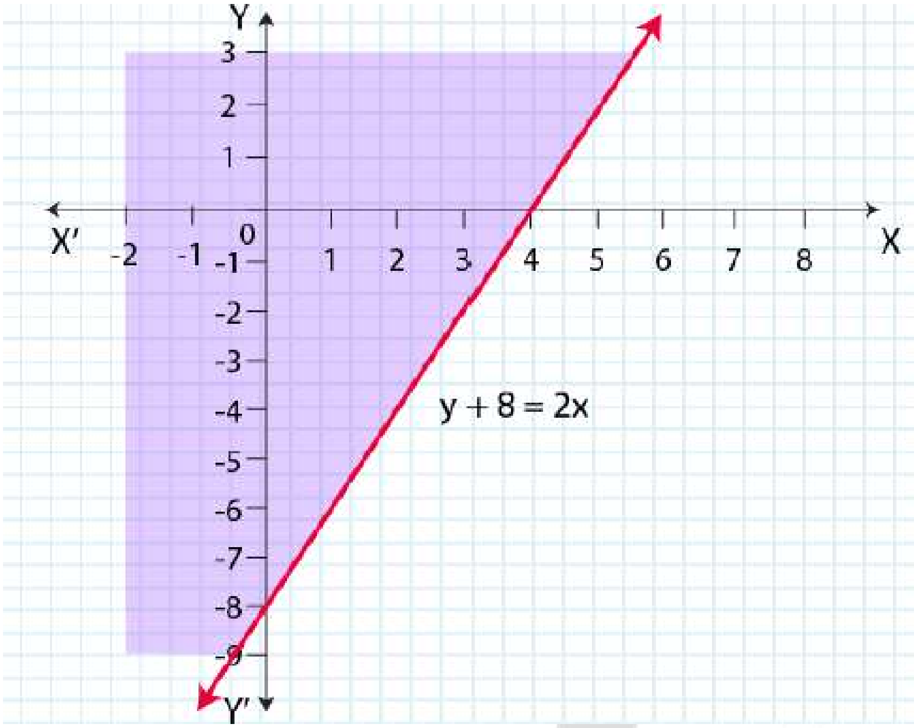
Select a point (0, 0)

$\Rightarrow (0) + 8 \geq 2 \times (0)$

$\Rightarrow 0 \leq 8$ (this is true)

$\therefore$ Solution region of the given inequality is above the line $y + 8 = 2x$. (i.e., origin is included in the region)

The graph is as follows:



5. $x - y \le 2$

Solution:

Given $x - y \le 2$

Now, draw a solid line $x - y = 2$ in the graph ($\because$ $x - y = 2$ is included in the given question).

Now, consider $x - y \le 2$

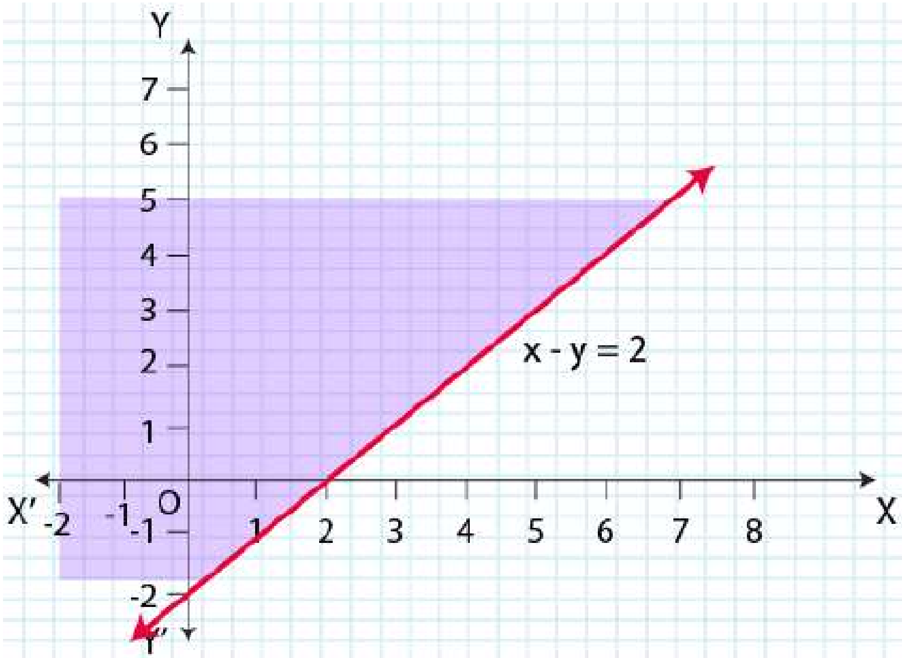
Select a point (0, 0)

$\Rightarrow (0) - (0) \le 2$

$\Rightarrow 0 \le 2$ (this is true)

$\therefore$ Solution region of the given inequality is above the line $x - y = 2$. (i.e., origin is included in the region)

The graph is as follows:



6. $2x - 3y > 6$

Solution:

Given $2x - 3y > 6$

Now draw a dotted line $2x - 3y = 6$ in the graph ($\because 2x - 3y = 6$ is excluded in the given question)

Now Consider $2x - 3y > 6$

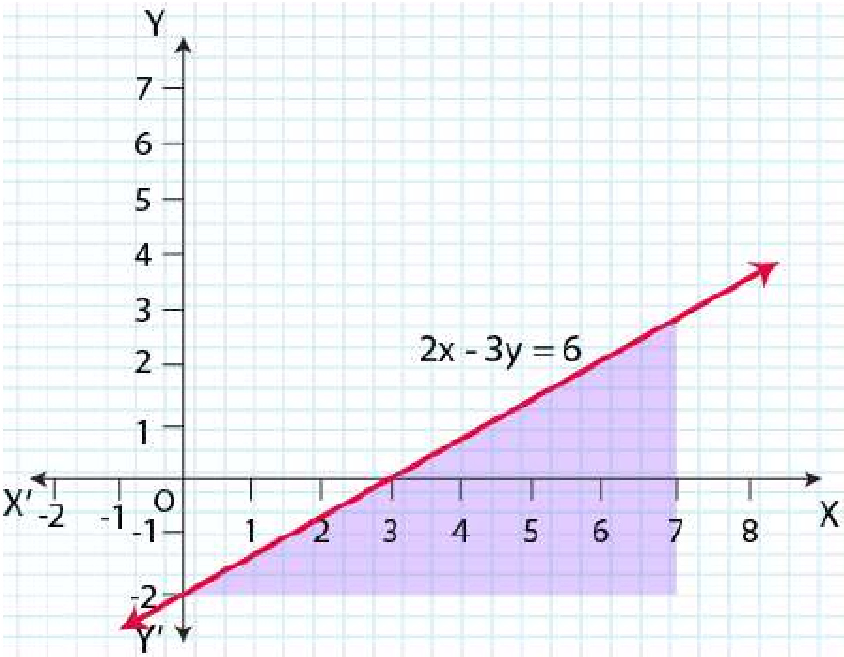
Select a point (0, 0)

$\Rightarrow 2 \times (0) - 3 \times (0) > 6$

$\Rightarrow 0 > 6$ (this is false)

$\therefore$ Solution region of the given inequality is below the line $2x - 3y > 6$. (Away from the origin)

The graph is as follows:



7. – 3x + 2y ≥ – 6

Solution:

Given – 3x + 2y ≥ – 6

Now, draw a solid line – 3x + 2y = – 6 in the graph (∵– 3x + 2y = – 6 is included in the given question).

Now, consider – 3x + 2y ≥ – 6

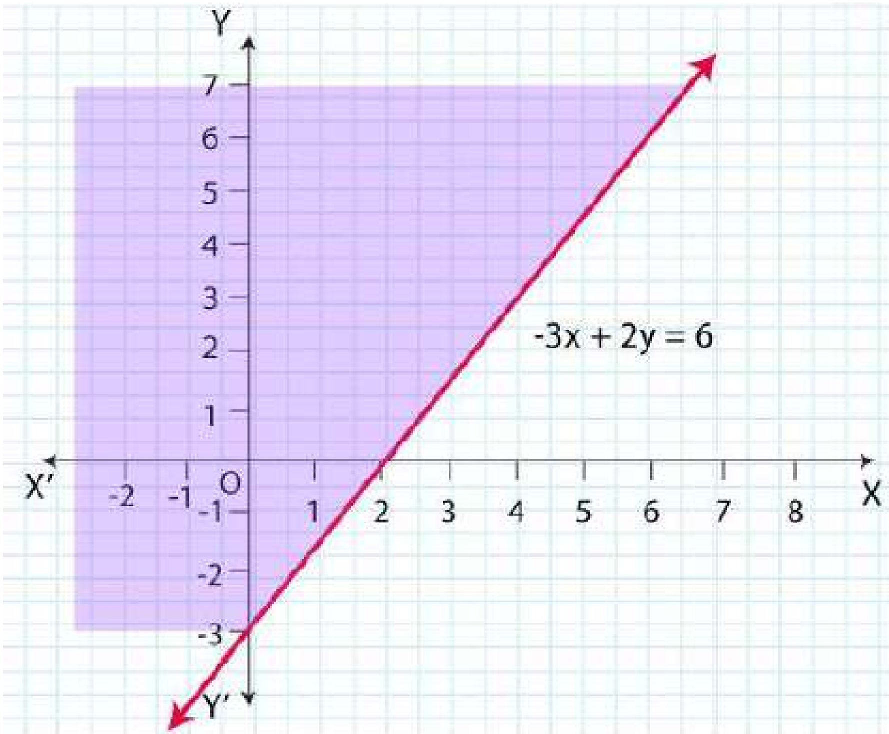
Select a point (0, 0)

⇒ – 3 × (0) + 2 × (0) ≥ – 6

⇒ 0 ≥ – 6 (this is true)

∴ Solution region of the given inequality is above the line – 3x + 2y ≥ – 6. (i.e., origin is included in the region)

The graph is as follows:



8. y – 5x < 30

Solution:

Given y – 5x < 30

Now, draw a dotted line 3y – 5x = 30 in the graph (∵3y – 5x = 30 is excluded in the given question)

Now, consider 3y – 5x < 30

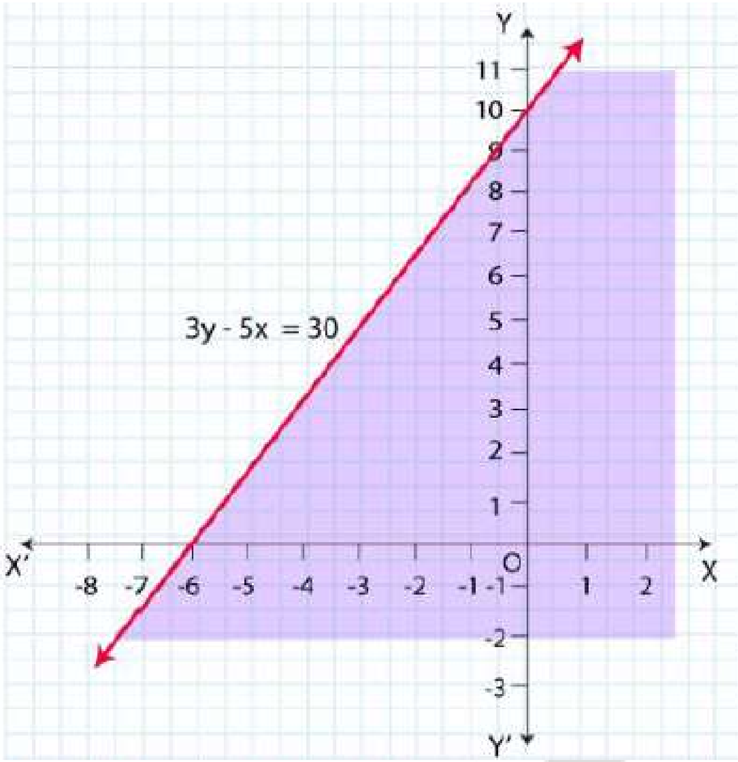
Select a point (0, 0)

⇒ 3 × (0) – 5 × (0) < 30

⇒ 0 < 30 (this is true)

∴ Solution region of the given inequality is below the line 3y – 5x < 30. (i.e., origin is included in the region)

The graph is as follows:



9. $y < -2$

Solution:

Given $y < -2$

Now, draw a dotted line $y = -2$ in the graph ($\because$ $y = -2$ is excluded in the given question)

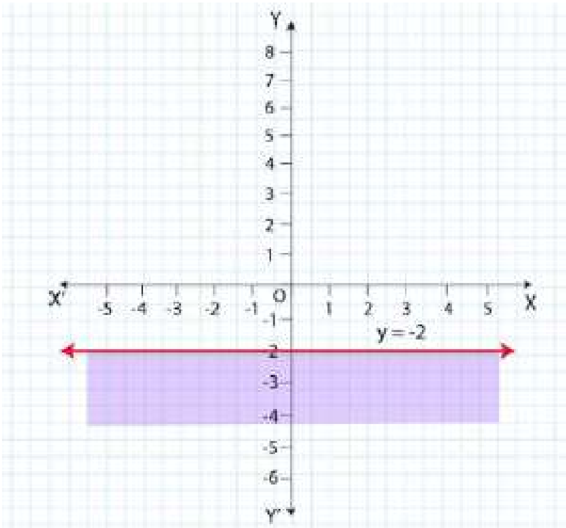
Now, consider $y < -2$

Select a point (0, 0)

$\Rightarrow 0 < -2$ (this is false)

$\therefore$ Solution region of the given inequality is below the line $y < -2$. (i.e., away from the origin)

The graph is as follows:



10. x > – 3

Solution:

Given x > – 3

Now, draw a dotted line x = – 3 in the graph (∵x = – 3 is excluded in the given question)

Now, consider x > – 3

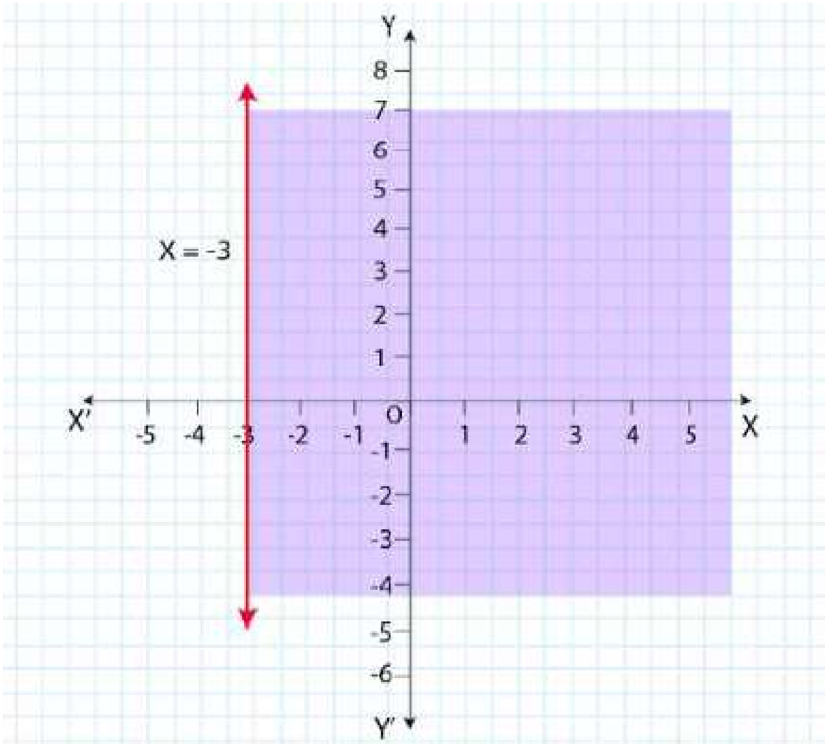
Select a point (0, 0)

⇒ 0 > – 3

⇒ 0 > – 3 (this is true)

∴ Solution region of the given inequality is right to the line x > – 3. (i.e., origin is included in the region)

The graph is as follows:



Exercise 6.3 Page No: 129

Solve the following system of inequalities graphically:

1. $x \geq 3, y \geq 2$

Solution:

Given $x \geq 3$……… (i)

$y \geq 2$…………… (ii)

Since $x \geq 3$ means for any value of y the equation will be unaffected, so similarly for $y \geq 2$, for any value of x the equation will be unaffected.

Now putting $x = 0$ in (i)

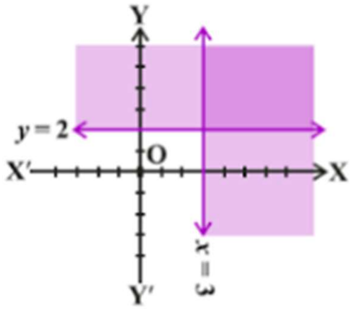
$0 \geq 3$ which is not true

Putting $y = 0$ in (ii)

$0 \geq 2$ which is not true again

This implies the origin doesn't satisfy in the given inequalities. The region to be included will be on the right side of the two equalities drawn on the graphs.

The shaded region is the desired region.



2. $3x + 2y \leq 12, x \geq 1, y \geq 2$

Solution:

Given $3x + 2y \leq 12$

Solving for the value of x and y by putting $x = 0$ and $y = 0$ one by one, we get $y = 6$ and $x = 4$

So the points are (0, 6) and (4, 0)

Now checking for (0, 0)

$0 \leq 12$ which is also true.

Hence, the origin lies in the plane and the required area is toward the left of the equation.

Now checking for $x \geq 1$, the value of x would be unaffected by any value of y. The origin would not lie on the plane.

$\Rightarrow 0 \geq 1$ which is not true

The required area to be included would be on the left of the graph $x \geq 1$

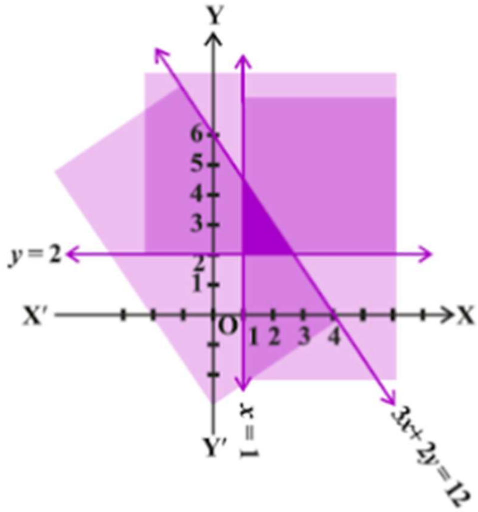
Similarly, for $y \geq 2$

Value of y will be unaffected by any value of x in the given equality. Also, the origin doesn't satisfy the given inequality.

$\Rightarrow 0 \geq 2$ which is not true. Hence origin is not included in the solution of the inequality.

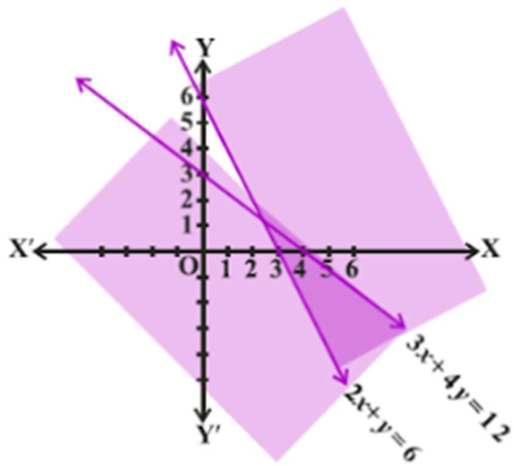
The region to be included in the solution would be towards the left of the equality $y \geq 2$

The shaded region in the graph will give the answer to the required inequalities as it is the region which is covered by all the given three inequalities at the same time satisfying all the given conditions.



3. $2x + y \geq 6$, $3x + 4y \leq 12$

Solution:
Given $2x + y \geq 6$…………… (i)
$3x + 4y \leq 12$ ……………. (ii)
$2x + y \geq 6$
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of
$y = 6$ and $x = 3$
So the point for the (0, 6) and (3, 0)
Now checking for (0, 0)
$0 \geq 6$ which is not true, hence the origin does not lie in the solution of the equality. The required region is on the right side of the graph.
Checking for $3x + 4y \leq 12$,
Putting value of $x= 0$ and $y = 0$ one by one in equation,
We get $y = 3$, $x = 4$
The points are (0, 3), (4, 0)
Now, checking for origin (0, 0)
$0 \leq 12$ which is true,
So the origin lies in solution of the equation.
The region on the right of the equation is the region required.
The solution is the region which is common to the graphs of both the inequalities.
The shaded region is the required region.



4. $x + y \geq 4, 2x - y < 0$

Solution:

Given $x + y \geq 4$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 4$ and $x = 4$

The points for the line are (0, 4) and (4, 0)

Checking for the origin (0, 0)

$0 \geq 4$

This is not true,

So the origin would not lie in the solution area. The required region would be on the right of line's graph.

$2x - y < 0$

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 0$ and $x = 0$

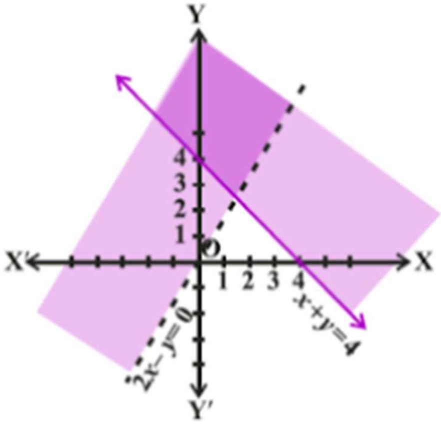
Putting $x = 1$ we get $y = 2$

So, the points for the given inequality are (0, 0) and (1, 2)

Now that the origin lies on the given equation, we will check for (4, 0) point to check which side of the line's graph will be included in the solution.

$\Rightarrow 8 < 0$ which is not true, hence the required region would be on the left side of the line $2x - y < 0$

The shaded region is the required solution of the inequalities.



5. $2x - y > 1, x - 2y < -1$

Solution:

Given $2x - y > 1$……………… (i)

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = -1$ and $x = 1/2 = 0.5$

The points are (0,-1) and (0.5, 0)

Checking for the origin, putting (0, 0)

$0 > 1$, which is false

Hence the origin does not lie in the solution region. The required region would be on the right of the line`s graph.

$x - 2y < -1$……….. (ii)

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = ½ = 0.5$ and $x = -1$

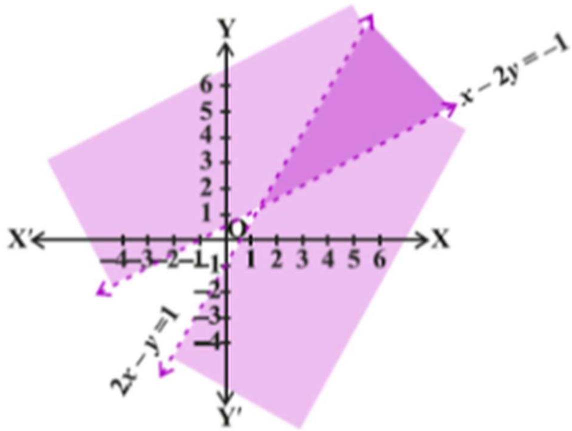
The required points are (0, 0.5) and (-1, 0)

Now checking for the origin, (0, 0)

$0 < -1$ which is false.

Hence, the origin does not lie in the solution area; the required area would be on the left side of the line's graph.

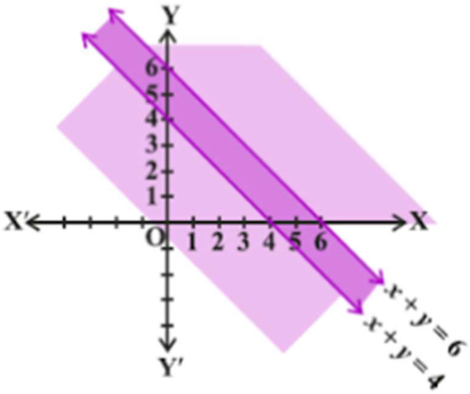
∴ The shaded area is the required solution of the given inequalities.



6. $x + y \leq 6, x + y \geq 4$

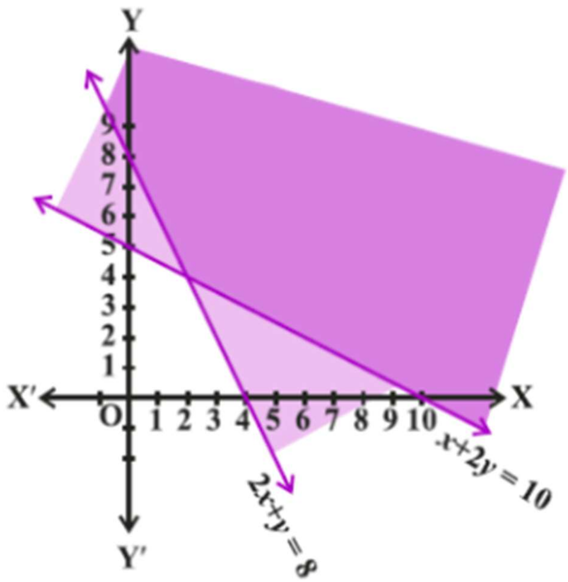
Solution:

Given $x + y \leq 6$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 6$ and $x = 6$
The required points are (0, 6) and (6, 0)
Checking further for origin (0, 0)
We get $0 \leq 6$, this is true.
Hence the origin would be included in the area of the line's graph. So, the required solution of the equation would be on the left side of the line graph which will be including origin.
$x + y \geq 4$
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 4$ and $x = 4$
The required points are (0, 4) and (4, 0)
Checking for the origin (0, 0)
$0 \geq 4$ which is false
So, the origin would not be included in the required area. The solution area will be above the line graph or the area on the right of line graph.
Hence, the shaded area in the graph is required graph area.



7. $2x + y \geq 8, x + 2y \geq 10$

Solution:
Given $2x + y \geq 8$
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of
$y = 8$ and $x = 4$
The required points are (0, 8) and (4, 0)
Checking if the origin is included in the line`s graph (0, 0)
$0 \geq 8$, which is false.
Hence, the origin is not included in the solution area and the required area would be the area to the right of the line's graph.
$x + 2y \geq 10$
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of
$y = 5$ and $x = 10$
The required points are (0, 5) and (10, 0)
Checking for the origin (0, 0)
$0 \geq 10$ which is false,
Hence the origin would not lie in the required solution area. The required area would be to the left of the line graph.
The shaded area in the graph is the required solution of the given inequalities.



8. $x + y \leq 9, y > x, x \geq 0$

Solution:

Given $x + y \leq 9$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 9$ and $x = 9$

The required points are (0, 9) and (9, 0)

Checking if the origin is included in the line's graph (0, 0)

$0 \leq 9$

Which is true. So, the required area would be including the origin and hence, will lie on the left side of the line`s graph.

$y > x$,

Solving for $y = x$

We get x= 0, y = 0, so the origin lies on the line's graph.

The other points would be (0, 0) and (2, 2)

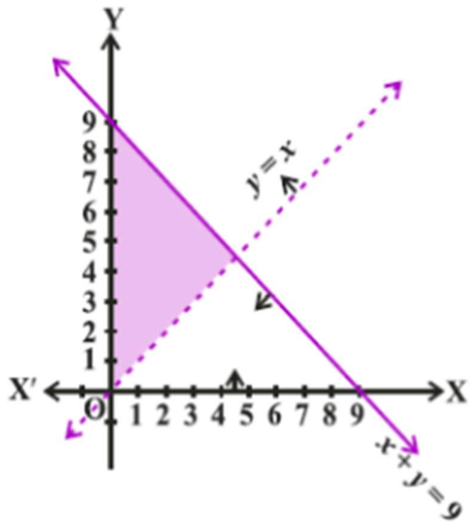
Checking for (9, 0) in $y > x$,

We get $0 > 9$ which is false, since the area would not include the area below the line's graph and hence, would be on the left side of the line.

We have $x \geq 0$

The area of the required line's graph would be on the right side of the line's graph.

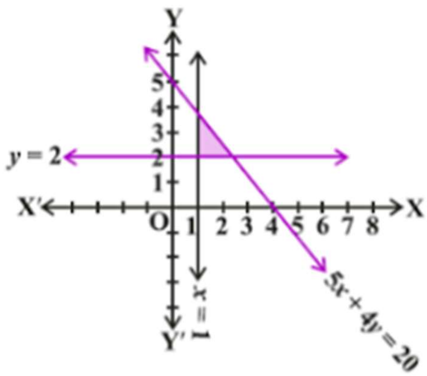
Therefore, the shaded area is the required solution of the given inequalities.



9. $5x + 4y \leq 20, x \geq 1, y \geq 2$

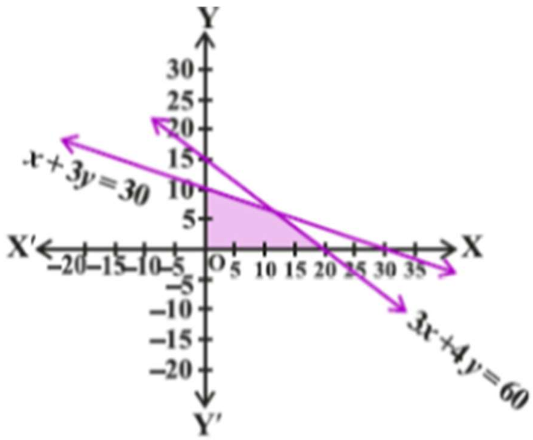
Solution:
Given $5x + 4y \leq 20$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 5$ and $x = 4$
The required points are (0, 5) and (4, 0)
Checking if the origin lies in the solution area (0, 0)
$0 \leq 20$
Which is true, hence the origin would lie in the solution area. The required area of the line's graph is on the left side of the graph.
We have $x \geq 1$,
For all the values of y, x would be 1,
The required points would be (1, 0), (1, 2) and so on.
Checking for origin (0, 0)
$0 \geq 1$, which is not true.
So, the origin would not lie in the required area. The required area on the graph will be on the right side of the line's graph.
Consider $y \geq 2$
Similarly for all the values of x, y would be 2.
The required points would be (0, 2), (1, 2) and so on.
Checking for origin (0, 0)
$0 \geq 2$, this is not true.
Hence, the required area would be on the right side of the line's graph.

The shaded area on the graph shows the required solution of the given inequalities.



10. $3x + 4y \leq 60, x + 3y \leq 30, x \geq 0, y \geq 0$

Solution:
Given $3x + 4y \leq 60$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of
$y = 15$ and $x = 20$
The required points are (0, 15) and (20, 0)
Checking if the origin lies in the required solution area (0, 0)
$0 \leq 60$, this is true.
Hence the origin would lie in the solution area of the line's graph.
The required solution area would be on the left of the line's graph.
We have $x + 3y \leq 30$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of
$y = 10$ and $x = 30$
The required points are (0, 10) and (30, 0).
Checking for the origin (0, 0)
$0 \leq 30$, this is true.
Hence the origin lies in the solution area which is given by the left side of the line's graph.
Consider $x \geq 0$,
$y \geq 0$,
The given inequalities imply the solution lies in the first quadrant only.
Hence the solution of the inequalities is given by the shaded region in the graph.

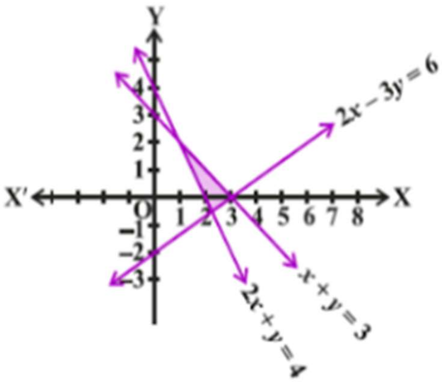


11. $2x + y \geq 4$, $x + y \leq 3$, $2x - 3y \leq 6$

Solution:

Given $2x + y \geq 4$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 4$ and $x = 2$
The required points are (0, 4) and (2, 0)
Checking for origin (0, 0)
$0 \geq 4$, this is not true
Hence the origin doesn't lie in the solution area of the line's graph. The solution area would be given by the right side of the line's graph.
$x + y \leq 3$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 3$ and $x = 3$
The required points are (0, 3) and (3, 0)
Checking for the origin (0, 0)
$0 \leq 3$, this is true.
Hence the solution area would include the origin and hence, would be on the left side of the line's graph.
$2x - 3y \leq 6$
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = -2$ and $x = 3$
The required points are (0, -2), (3, 0).
Checking for the origin (0, 0)
$0 \leq 6$ this is true
So the origin lies in the solution area and the area would be on the left of the line's graph.

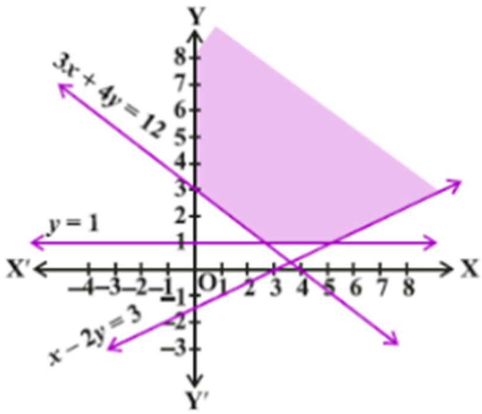
Hence, the shaded area in the graph is the required solution area for the given inequalities.



12. $x - 2y \leq 3$, $3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$

Solution:
Given, $x - 2y \leq 3$
Putting value of $x = 0$ and $y = 0$ in the equation one by one, we get value of $y = -3/2 = -1.5$ and $x = 3$
The required points are (0, -1.5) and (3, 0)
Checking for the origin (0, 0)
$0 \leq 3$, this is true.
Hence, the solution area would be on the left of the line's graph
$3x + 4y \geq 12$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 3$ and $x = 4$
The required points are (0, 3) and (4, 0)
Checking for the origin (0, 0)
$0 \geq 12$, this is not true.
So, the solution area would include the origin and the required solution area would be on the right side of the line's graph.
We have $x \geq 0$,
For all the values of y, the value of x would be same in the given inequality, which would be the region above the x axis on the graph.
Consider, $y \geq 1$
For all the values of x, the value of y would be same in the given inequality.
The solution area of the line would not include origin as $0 \geq 1$ is not true.
The solution area would be on the left side of the line's graph.

The shaded area in the graph is the required solution area which satisfies all the given inequalities at the same time.



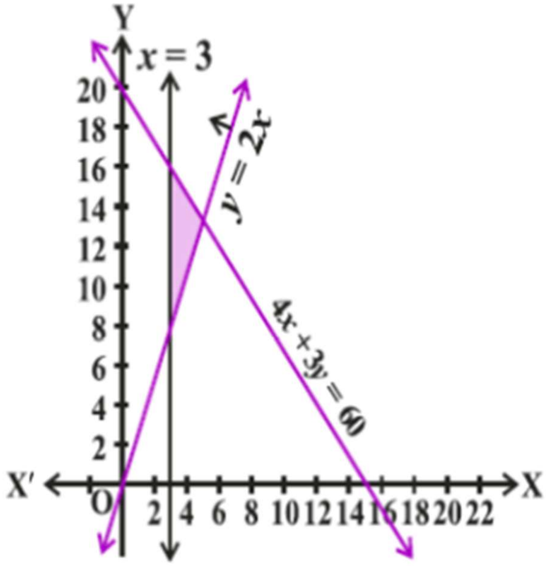
13. $4x + 3y \leq 60, y \geq 2x, x \geq 3, x, y \geq 0$

Solution:
Given, $4x + 3y \leq 60$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of
$y = 20$ and $x = 15$
The required points are (0, 20) and (15, 0).
Checking for the origin (0, 0)
$0 \leq 60$, this is true.
Hence the origin would lie in the solution area. The required area would be on the left of the line's graph.
We have $y \geq 2x$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of
$y = 0$ and $x = 0$
Hence the line would pass through origin.
To check which side would be included in the line's graph solution area, we would check for point (15, 0)
$\Rightarrow 0 \geq 15$, this is not true. So the required solution area would be to the left of the line's graph.
Consider, $x \geq 3$,
For any value of y, the value of x would be same.
Also the origin (0, 0) doesn't satisfiy the inequality as $0 \geq 3$.
So, the origin doesn't lie in the solution area. Hence, the required solution area would be on the right of the line's graph.
We have $x, y \geq 0$

Since it is given both x and y are greater than 0

∴ the solution area would be in the first Ist quadrant only.

The shaded area in the graph shows the solution area for the given inequalities.



14. $3x + 2y \leq 150$, $x + 4y \leq 80$, $x \leq 15$, $y \geq 0$, $x \geq 0$

Solution:

Given, $3x + 2y \leq 150$

Putting value of $x = 0$ and $y = 0$ in the equation one by one, we get value of $y = 75$ and $x = 50$

The required points are (0, 75) and (50, 0).

Checking for the origin (0, 0)

$0 \leq 150$, this is true.

Hence, the solution area for the line would be on the left side of the line's graph, which would be including the origin too.

We have $x + 4y \leq 80$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 20$ and $x = 80$

The required points are (0, 20) and (80, 0).

Checking for the origin (0, 0)

$0 \leq 80$, this is also true. So, the origin lies in the solution area.

The required solution area would be toward the left of the line's graph.

Given $x \leq 15$,

For all the values of y, x would be same.

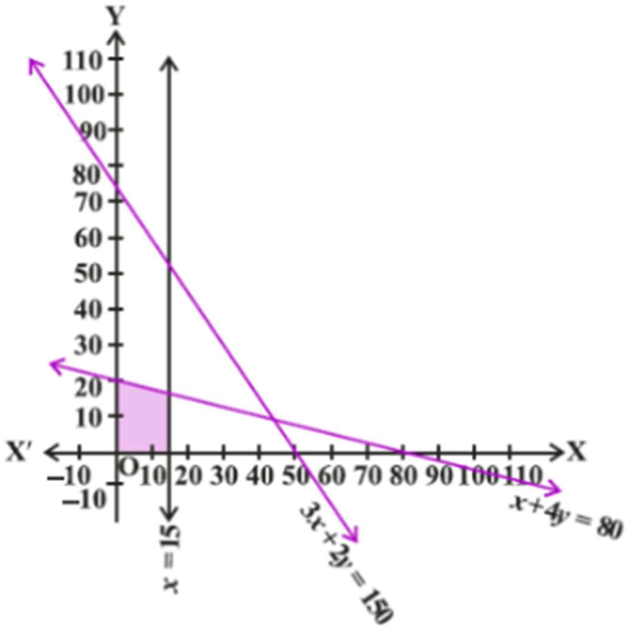
Checking for the origin (0, 0)

$0 \leq 15$, this is true. So, the origin would be included in the solution area. The required solution area would be towards the left of the line's graph.

Consider $y \geq 0$, $x \geq 0$

Since x and y are greater than 0, the solution would lie in the 1st quadrant.

The shaded area in the graph satisfies all the given inequalities, and hence is the solution area for given inequalities.



15. $x + 2y \leq 10$, $x + y \geq 1$, $x - y \leq 0$, $x \geq 0$, $y \geq 0$

Solution:

Given, $x + 2y \leq 10$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 5$ and $x = 10$

The required points are (0, 5) and (10, 0).

Checking for the origin (0, 0)

$0 \leq 10$, this is true.

Hence, the solution area would be toward origin including the same. The solution area would be toward the left of the line's graph.

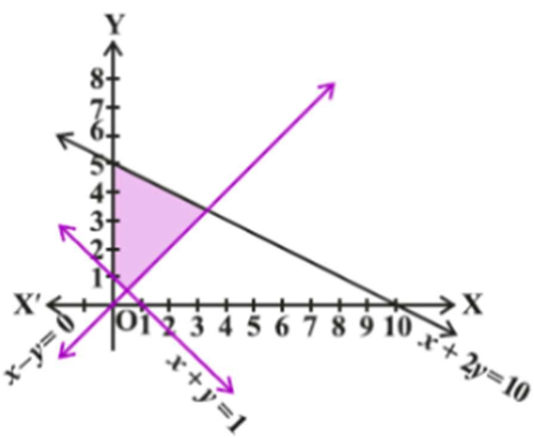
We have $x + y \geq 1$,

Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of $y = 1$ and $x = 1$

The required points are (0, 1) and (1, 0)

Checking for the origin (0, 0)

$0 \geq 1$, this is not true.
Hence, the origin would not be included in the solution area. The required solution area would be toward the right of the line's graph.
Consider $x - y \leq 0$,
Putting value of $x = 0$ and $y = 0$ in equation one by one, we get value of
$y = 0$ and $x = 0$
Hence, the origin would lie on the line.
To check which side of the line graph would be included in the solution area, we would check for the (10, 0)
$10 \leq 0$, which is not true. Hence, the solution area would be on the left side of the line's graph.
Again, we have $x \geq 0$, $y \geq 0$
Since both x and y are greater than 0, the solution area would be in the 1^{st} quadrant.
Hence, the solution area for the given inequalities would be the shaded area of the graph satisfying all the given inequalities.



Miscellaneous Exercise Page No: 129

Solve the inequalities in Exercises 1 to 6
1. $2 \leq 3x - 4 \leq 5$
Solution:
According to the question,
The inequality given is,
$2 \leq 3x - 4 \leq 5$
$\Rightarrow 2 \leq 3x - 4 \leq 5$
$\Rightarrow 2 + 4 \leq 3x - 4 + 4 \leq 5 + 4$
$\Rightarrow 6 \leq 3x \leq 9$

⇒ $6/3 \le 3x/3 \le 9/3$
⇒ $2 \le x \le 3$
Hence, all real numbers x greater than or equal to 2, but less than or equal to 3 are solutions of given equality.
$x \in [2, 3]$

2. $6 \le -3(2x - 4) < 12$

Solution:

According to the question,
The inequality given is,
$6 \le -3(2x - 4) < 12$
⇒ $6 \le -3(2x - 4) < 12$
Dividing the inequality by 3, we get.
⇒ $2 \le -(2x - 4) < 4$
Multiplying the inequality by -1,
⇒ $-2 \ge 2x - 4 > -4$ [multiplying the inequality with -1 changes the inequality sign.]
⇒ $-2 + 4 \ge 2x - 4 + 4 > -4 + 4$
⇒ $2 \ge 2x > 0$
Dividing the inequality by 2,
⇒ $0 < x \le 1$
Hence, all real numbers x greater than 0, but less than or equal to 1 are solutions of given equality.
$x \in (0, 1]$

3. $-3 \le 4 - 7x/2 \le 18$

Solution:

According to the question,
The inequality given is,
$-3 \le 4 - 7x/2 \le 18$
⇒ $-3 - 4 \le 4 - 7x/2 - 4 \le 18 - 4$
⇒ $-7 \le -7x/2 \le 18 - 14$
Multiplying the inequality by -2,

$$\Rightarrow (-7) \times (-2) \ge -\frac{7x}{2} \times (-2) \ge 14 \times (-2)$$

⇒ $14 \ge 7x \ge -28$
⇒ $-28 \le 7x \le 14$
Dividing the inequality by 7,
⇒ $-4 \le x \le 2$

Hence, all real numbers x greater than or equal to -4, but less than or equal to 2 are solutions of given equality.

$x \in [-4, 2]$

4. $-15 \leq 3(x-2)/5 \leq 0$

Solution:

According to the question,

The inequality given is,

$-15 \leq 3(x-2)/5 \leq 0$

$\Rightarrow -15 < 3(x-2)/5 \leq 0$

Multiplying the inequality by 5,

$$\Rightarrow -15 \times 5 < \frac{3(x-2)}{5} \times 5 \leq 0 \times 5$$

$\Rightarrow -75 < 3(x-2) \leq 0$

Dividing the inequality by 3, we get,

$$\Rightarrow -\frac{75}{3} < \frac{3(x-2)}{3} \leq \frac{0}{3}$$

$\Rightarrow -25 < x-2 \leq 0$

$\Rightarrow -25 + 2 < x - 2 + 2 \leq 0 + 2$

$\Rightarrow -23 < x \leq 2$

Hence, all real numbers x greater than -23, but less than or equal to 2 are solutions of given equality.

$x \in (-23, 2]$

5. $-12 < 4 - 3x/(-5) \leq 2$

Solution:

According to the question,

The inequality given is,

$-12 < 4 - \frac{3x}{-5} \leq 2$

$\Rightarrow -12 < 4 - \frac{3x}{-5} \leq 2$

$\Rightarrow -12 - 4 < 4 - \frac{3x}{-5} - 4 \leq 2 - 4$

$\Rightarrow -16 < \frac{-3x}{-5} \leq -2$

$\Rightarrow -16 < \frac{3x}{5} \leq -2$

Multiplying the inequality by 5.

$\Rightarrow -16 \times 5 < \frac{3x}{5} \times 5 \leq -2 \times 5$

$\Rightarrow -80 < 3x \leq -10$

$\Rightarrow -\frac{80}{3} < x \leq -\frac{10}{3}$

Hence, all real numbers x greater than -80/3, but less than or equal to -10/3 are solutions of given equality.

x ∈ (-80/3, -10/3]

6. $7 \leq (3x + 11)/2 \leq 11$

Solution:

According to the question,

The inequality given is,

$7 \leq \frac{(3x+11)}{2} \leq 11$

$\Rightarrow 7 \leq \frac{(3x+11)}{2} \leq 11$

Multiplying the inequality by 2.

$\Rightarrow 7 \times 2 \leq \frac{(3x+11)}{2} \times 2 \leq 11 \times 2$

$\Rightarrow 14 \leq 3x + 11 \leq 22$

$\Rightarrow 14 - 11 \leq 3x + 11 - 11 \leq 22 - 11$

$\Rightarrow 3 \leq 3x \leq 11$

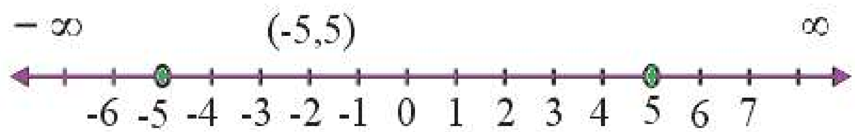
$\Rightarrow 1 \leq x \leq 11/3$

Hence, all real numbers x greater than or equal to -4, but less than or equal to 2 are solutions of given equality.

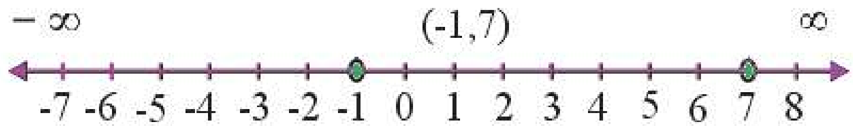
x ∈ [1, 11/3]

Solve the inequalities in Exercises 7 to 11 and represent the solution graphically on number line.

7. 5x + 1 > – 24, 5x – 1 < 24
Solution:
According to the question,
The inequalities given are,
5x + 1 > -24 and 5x – 1 < 24
5x + 1 > -24
⇒ 5x > -24 – 1
⇒ 5x > -25
⇒ x > -5 ……… (i)
5x – 1 < 24
⇒ 5x < 24 + 1
⇒ 5x < 25
⇒ x < 5 ……….(ii)
From equations (i) and (ii),
We can infer that the solution of given inequalities is (-5, 5).



8. 2 (x – 1) < x + 5, 3 (x + 2) > 2 – x
Solution:
According to the question,
The inequalities given are,
2 (x – 1) < x + 5 and 3 (x + 2) > 2 – x
2 (x – 1) < x + 5
⇒ 2x – 2 < x + 5
⇒ 2x – x < 5 + 2
⇒ x < 7 ……… (i)
3 (x + 2) > 2 – x
⇒ 3x + 6 > 2 – x
⇒ 3x + x > 2 – 6
⇒ 4x > -4
⇒ x > -1 ………. (ii)
From equations (i) and (ii),
We can infer that the solution of given inequalities is (-1, 7).



9. $3x - 7 > 2(x - 6)$, $6 - x > 11 - 2x$

Solution:

According to the question,

The inequalities given are,

$3x - 7 > 2(x - 6)$ and $6 - x > 11 - 2x$

$3x - 7 > 2(x - 6)$

$\Rightarrow 3x - 7 > 2x - 12$

$\Rightarrow 3x - 2x > 7 - 12$

$\Rightarrow x > -5$ ………… (i)

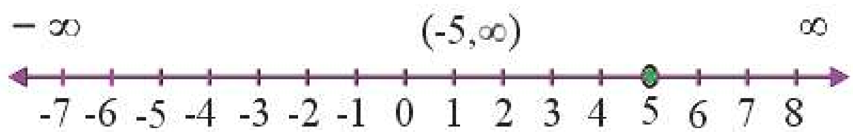
$6 - x > 11 - 2x$

$\Rightarrow 2x - x > 11 - 6$

$\Rightarrow x > 5$ ……….(ii)

From equations (i) and (ii),

We can infer that the solution of given inequalities is $(5, \infty)$.



10. $5(2x - 7) - 3(2x + 3) \leq 0$, $2x + 19 \leq 6x + 47$

Solution:

According to the question,

The inequalities given are,

$5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 47$

$5(2x - 7) - 3(2x + 3) \leq 0$

$\Rightarrow 10x - 35 - 6x - 9 \leq 0$

$\Rightarrow 4x - 44 \leq 0$

$\Rightarrow 4x \leq 44$

$\Rightarrow x \leq 11$ ……(i)

$2x + 19 \leq 6x + 47$

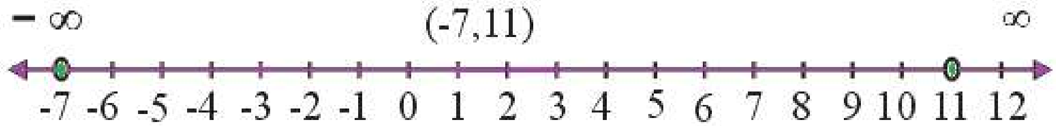
$\Rightarrow 6x - 2x \geq 19 - 47$

$\Rightarrow 4x \geq -28$

$\Rightarrow x \geq -7$ ……….(ii)

From equations (i) and (ii),

We can infer that the solution of given inequalities is (-7, 11).



11. A solution is to be kept between 68° F and 77° F. What is the range in temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) conversion formula is given by F = (9/5) C + 32?

Solution:

According to the question,

The solution has to be kept between 68° F and 77° F

So, we get, $68° < F < 77°$

Substituting,

$$F = \frac{9}{5}C + 32$$

$$\Rightarrow 68 < \frac{9}{5}C + 32 < 77$$

$$\Rightarrow 68 - 32 < \frac{9}{5}C + 32 - 32 < 77 - 32$$

$$\Rightarrow 36 < \frac{9}{5}C < 45$$

$$\Rightarrow 36 \times \frac{5}{9} < \frac{9}{5}C \times \frac{5}{9} < 45 \times \frac{5}{9}$$

$$\Rightarrow 20 < C < 25$$

Hence, we get,

The range of temperature in degree Celsius is between 20° C to 25° C.

12. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4%, but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Solution:

According to the question,

8% of solution of boric acid = 640 litres

Let the amount of 2% boric acid solution added = x litres

Then we have,

Total mixture = x + 640 litres

We know that,

The resulting mixture has to be more than 4% but less than 6% boric acid.

∴ 2% of x + 8% of 640 > 4% of (x + 640) and

2% of x + 8% of 640 < 6% of (x + 640)

2% of x + 8% of 640 > 4% of (x + 640)

⇒ (2/100) × x + (8/100) × 640 > (4/100) × (x + 640)

⇒ 2x + 5120 > 4x + 2560

⇒ 5120 – 2560 > 4x – 2x

⇒ 2560 > 2x

⇒ x < 1280 ….(i)

2% of x + 8% of 640 < 6% of (x + 640)

⇒ (2/100) × x + (8/100) × 640 < (6/100) × (x + 640)

⇒ 2x + 5120 < 6x + 3840

⇒ 6x – 2x > 5120 – 3840

⇒ 4x > 1280

⇒ x > 320 ……….(i)

From (i) and (ii)

320 < x < 1280

Therefore, the number of litres of 2% of boric acid solution that has to be added will be more than 320 litres but less than 1280 litres.

13. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Solution:

According to the question,

45% of solution of acid = 1125 litres

Let the amount of water added = x litres

Resulting mixture = x + 1125 litres

We know that,

The resulting mixture has to be more than 25% but less than 30% acid content.

Amount of acid in resulting mixture = 45% of 1125 litres.

∴ 45% of 1125 < 30% of (x + 1125) and 45% of 1125 > 25% of (x + 1125)

45% of 1125 < 30% of (x + 1125)

$\Rightarrow \frac{45}{100} \times 1125 < \frac{30}{100} \times (x + 1125)$

⇒ 45 × 1125 < 30x + 30 × 1125

⇒ (45 – 30) × 1125 < 30x

⇒ 15 × 1125 < 30x

⇒ x > 562.5 ………..(i)

45% of 1125 > 25% of (x + 1125)

$\Rightarrow \frac{45}{100} \times 1125 > \frac{25}{100} \times (x + 1125)$

$\Rightarrow 45 \times 1125 > 25x + 25 \times 1125$

$\Rightarrow (45 - 25) \times 1125 > 25x$

$\Rightarrow 25x < 20 \times 1125$

$\Rightarrow x < 900$ .....(ii)

$\therefore 562.5 < x < 900$

Therefore, the number of litres of water that has to be added will have to be more than 562.5 litres but less than 900 litres.

14. IQ of a person is given by the formula $IQ = \frac{MA}{CA} \times 100$, Where MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12-year-old children, find the range of their mental age.

Solution:

According to the question,

Chronological age = CA = 12 years

IQ for age group of 12 is $80 \leq IQ \leq 140$.

We get that,

$80 \leq IQ \leq 140$

Substituting,

$IQ = \frac{MA}{CA} \times 100$

We get,

$\Rightarrow 80 \leq \frac{MA}{CA} \times 100 \leq 140$

$\Rightarrow 80 \leq \frac{MA}{12} \times 100 \leq 140$

$\Rightarrow 80 \times \frac{12}{100} \leq \frac{MA}{12} \times 100 \leq 140 \times \frac{12}{100}$

$\Rightarrow 9.6 \leq MA \leq 16.8$

$\therefore$ Range of mental age of the group of 12 year-old-children is $9.6 \leq MA \leq 16.8$

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